XV.—*On the theoretical investigation of the velocity of sound, as corrected from M.* Dulong's *recent experiments, compared with the results of the observations of Dr.* Moll *and Dr.* Van Beek. *By Dr.* Simons, *Assistant at the Observatory of the University of Utrecht. Communicated by Captain* Henry Kater, *Vice-President.*

Read March 18, 1830.

It has been demonstrated by the ever-to-be-lamented Laplace*, that in order to obtain the velocity of sound by calculation, Sir Isaac Newton's original expression†,

$$V = \sqrt{\frac{g \cdot p}{D}},$$

must be multiplied by the square root of the ratio between the specific heats of atmospheric air under a constant pressure and under a constant volume. In this formula V is the velocity of sound, g the intensity of gravitating force, p the atmospheric pressure, and D the density of the medium through which sound is transmitted; the density of mercury being equal to 1.

The coefficient, which is to multiply the Newtonian formula, has been deduced by M. Laplace, first from MM. Laroche and Berardo's‡ experiments, next from those of MM. Clement and Desormes§, and finally from the more accurate investigations of MM. Gay-Lussac and Welter.

By introducing this correction, the velocity as deduced from calculation, was found to differ but little from what is actually obtained by experiment. But this difference between calculation and experiment, however small, was always such, that the observed velocity constantly exceeded that which was deduced by calculation.

* Annales des Physique et de Chimie, t. iii. p. 238. t. xxiii. p. 1. Mécan. Céleste, t. v. p. 119, *seqq.*

† Princip. l. ii. prop. 48.

‡ Annales de Chimie, t. lxxxv. p. 72.

§ Journal de Physique, t. lxxxix. p. 333, *seqq.*

The degree of accuracy with which experiments on the velocity of sound are now conducted, naturally led to a supposition that some of the elements of the theoretical formula were susceptible of a greater degree of correctness; and thus natural philosophers were rather inclined to attribute the difference between experiment and calculation to some deficiency of the analytical expression, than to error in the observation.

It has been shown in a recent volume of the Philosophical Transactions, that the experiments on the velocity of sound, made by Captain PARRY in the polar regions, lead to the same conclusions with those made by Drs. MOLL and VAN BEEK, under widely different circumstances; and this coincidence would tend to confirm our doubts as to the correctness of some of the elements of the computation.

A very able experimentalist, M. DULONG, lately published some experiments on the specific heat of the gases*, in the investigation of which he recurs to LAPLACE's analytical formula. M. DULONG's reasoning is nearly this: If it be admitted that the velocity of sound in atmospheric air is obtained by multiplying the Newtonian formula by the square root of the ratio between the two specific heats of the air, under a constant pressure, and under a constant volume, it must follow, that this ratio, or the coefficient with whose square root the original formula must be multiplied, may also be deduced from the velocity of sound as given by observation. Accordingly, M. DULONG computed this ratio or coefficient from actual observations on the velocity of sound, by the formula

$$K = \frac{V^2}{\frac{g \cdot p}{D}}$$

in which K is the ratio between the specific heat of air under a constant pressure, and the specific heat of air under a constant volume, whilst V is the velocity of sound as obtained by experiment.

The object of this paper is to compare the inquiries of M. DULONG with the experiments on the velocity of sound made by Drs. MOLL and VAN BEEK, which were published in the Philosophical Transactions.

The following Table contains only such of the observations of Drs. MOLL and VAN BEEK, in which the guns were fired on both stations exactly at the same

* Annales de Physique et de Chimie, t. xli. p. 113.

moment, and in which the interval of time between the flash and the report was correctly observed at both extremities of the base.

Date.	No. of the Observation.	Barometrical Pressure reduced to 0° Centigr. and corrected for capillarity. Metres *p*.	Tension of Aqueous Vapour in Metres T.	Temperature, Centigrade Thermometer. *t*.	Time elapsed between Flash and Report.
1823.				°	seconds.
June 27	1	0.74450	0.0095347	12.08	52.035
	3	0.74455	0.0095474	12.08	51.790
	4	0.74455	0.0094585	12.01	51.695
	5	0.74465	0.0096998	11.94	51.860
	6	0.74455	0.0095347	11.88	51.850
	8	0.74470	0.0094459	11.67	51.950
	9	0.74470	0.0093697	11.60	51.845
	12	0.74475	0.0093697	11.25	51.865
	13	0.74470	0.0092173	11.25	51.945
	17	0.74500	0.0093697	10.47	51.960
	18	0.74505	0.0092173	10.40	52.055
	19	0.74495	0.0092173	10.40	52.025
	23	0.74480	0.0089888	10.28	51.995
	24	0.74470	0.0088364	10.14	51.945
	25	0.74475	0.0088364	10.07	52.020
	26	0.74480	0.0084683	10.00	51.335
June 28	4	0.74830	0.0084638	11.89	52.020
	5	0.74775	0.0083921	11.09	51.525
	6	0.74800	0.0084638	11.53	52.245
	7	0.74820	0.0081763	11.25	32.315
	8	0.74815	0.0081763	10.78	52.215
	9	0.74815	0.0083159	10.42	52.675
	10	0.74835	0.0083921	10.28	51.175
	14	0.74810	0.0081763	10.14	52.445
	15	0.74810	0.0082524	10.42	51.965
	17	0.74815	0.0086206	10.97	51.875
	18	0.74805	0.0086968	10.89	52.240
	19	0.74805	0.0087730	11.12	52.080

Length of base, 17669 metres.

From this Table and the following data, the results obtained by M. Dulong were compared with Drs. Moll and Van Beek's experiments.

The weight of a cube centimetre of mercury at the temperature of 0° Centigrade, is 13.596152 grammes. This is according to the experiments of MM. Biot and Arago, and MM. Dulong and Petit's subsequent investigations. The weight of a cube centimetre of dry atmospheric air, at zero of Centigrade scale, and under the barometric pressure of $0^{m}.760$ is, according to the same observers, 0.001299541 grammes*.

* Biot, Traité de Physiq. Expér. et Mathém. t. i. p. 402, *seqq*. and p. 387.

Now, if the intensity of the gravitating force at Paris be g', whilst it is g at the place where the experiments are made, we have the weight of a cube centimetre of dry atmospheric air, under a pressure of $760mm$, and at zero Centigrade, $0.001299541 \frac{g}{g'}$.

Under the same circumstances the ratio of the density of air to that of mercury, will be:

$$D = \frac{0.001299541}{13.596152} \cdot \frac{g}{g'} = \frac{1}{10462.273} \cdot \frac{g}{g'}.$$

If the barometric pressure becomes p, the temperature t Centigrade, the tension of aqueous vapor T, the density of the air becomes

$$D' = \frac{p - \frac{3}{8}T}{10462.273 \times 0.760 \{1 + 0.00375 . t\}} \cdot \frac{g}{g}$$

substituting this value of D in the formula of the velocity of Sound, and observing that $g = g' \cdot \frac{g}{g'}$ we have,

$$V = \sqrt{\frac{10462.273 \times 0.760 \{1 + 0.00375 . t.\} p . g' . \frac{g}{g'}}{\{p - \frac{3}{8}T\} . \frac{g}{g'},}} \times \sqrt{K.}$$

or, $$V = \sqrt{10462.273 \times 0.760 \{1 + 0.00375 . t.\} \frac{g' . p}{p - \frac{3}{8}T}} \times \sqrt{K.}$$

This formula shows that V is independent of the latitude; and thus that the velocity of sound is not directly affected by the geographical position of the place of observation. From the late M. Borda's pendulum experiments, we have the intensity of gravity at Paris, $g' = 9.82827$.

From MM. Gay-Lussac and Welter's experiments, the value of K is deduced $K = 1.3748$.

From the more recent investigations of M. Dulong, we have $K = 1.421$.

Thus, taking K from MM. Gay-Lussac and Welter, the velocity of sound is

$$V = \sqrt{9.82827 \times 10462.273 \times 0.760 \{1 + 0.00375 . t.\} \frac{p}{p - \frac{3}{8}T}} \times \sqrt{1.3748}$$

and taking K from M. Dulong,

$$V = \sqrt{9.82827 \times 10642.273 \times 0.760 \{1 + 0.00375 . t.\} \frac{p}{p. - \frac{3}{8}T}} \times \sqrt{1.421}$$

Finally, supposing V′ the velocity of sound observed at a temperature *t* and a tension T, and V″ the velocity of sound at zero Centigrade and dry air, we have

$$V'' = V' \times \sqrt{\frac{p - \frac{3}{8}T}{p\{t + 0.00375 . t\}}}$$

From these formulæ the experiments of Messrs. Moll and Van Beek are calculated; the results of which are contained in the following Table.

A Comparative Table of the Velocity of Sound, as deduced by calculation, and obtained by the experiments of Drs. Moll and Van Beek.

Date.	Number of experiments.	Veloc. calc. from the determination of K by MM. Gay-Lussac & Welter in 1″.	Velocity observed by Drs. Moll and Van Beek in 1″.	Diff. of Obs. and Calc. Veloc., taking K from MM. Gay-Lussac and Welter.	Velocity calculated from the determination of K by M. Dulong in 1″.	Difference of Observed and Calculated Velocity taking K from M. Dulong.	Value of K as deduced from the experiments of Drs. Moll and Van Beek.	Observed Velocity reduced to 0° Cent. and in dry air.	Diff. between the observed reduced Velocity, and the mean reduced Velocity.
1823.		m	m	m	m	m		m	m
June27	1	335.590	339.565	+ 5.025	341.182	−1.617	1.4065	331.327	−0.917
	3	339.477	341.172	1.695	345.133	−3.961	1.3885	329.083	−3.161
	4	335.599	341.799	6.200	341.192	+0.607	1.4260	333.497	+1.253
	5	335.519	340.711	5.192	341.110	−0.399	1.4176	332.515	+0.271
	6	335.469	340.777	5.309	341.059	−0.282	1.4186	332.629	+0.385
	8	334.818	340.721	5.303	340.397	−0.276	1.4187	332.634	+0.390
	9	335.287	340.810	5.523	340.873	−0.063	1.4204	332.842	+0.598
	12	335.075	340.678	5.603	340.659	+0.019	1.4211	332.924	+0.680
	13	334.292	340.154	5.862	340.646	−0.492	1.4169	332.424	+0.180
	17	334.604	340.055	5.451	340.180	−0.125	1.4199	332.783	+0.539
	18	334.527	339.435	4.908	340.101	−0.666	1.4154	332.252	+0.008
	19	334.527	339.631	5.104	340.101	−0.470	1.4170	332.444	+0.200
	23	334.453	339.827	5.374	340.026	−0.199	1.4193	332.709	+0.465
	24	334.360	340.154	5.794	339.932	+0.222	1.4228	333.122	+0.878
	25	333.933	339.663	5.730	339.498	+0.165	1.4224	333.067	+0.823
	26	334.245	340.219	5.974	339.814	+0.405	1.4243	333.301	+1.057
June28	4	335.358	339.663	4.305	340.946	−1.283	1.4103	331.652	−0.592
	5	334.893	342.927	8.034	340.474	+2.453	1.4415	335.032	+2.788
	6	335.164	338.200	3.036	340.749	−2.549	1.3998	330.605	−1.539
	7	334.971	337.554	2.583	340.553	−2.999	1.3961	329.973	−2.271
	8	334.688	338.395	3.707	340.262	−1.870	1.4053	331.075	−1.119
	9	334.482	335.440	0.958	340.056	−4.616	1.3827	328.386	−3.858
	10	334.404	345.272	10.868	339.976	+5.296	1.4622	338.090	+6.154
	14	334.301	336.911	2.610	339.872	−2.961	1.3963	330.004	−2.240
	15	334.481	340.023	5.542	340.054	−0.031	1.4207	332.874	+0.630
	17	334.840	340.613	5.773	340.419	+0.194	1.4226	333.094	+0.850
	18	334.798	338.233	3.435	340.377	−2.144	1.4031	330.808	−1.436
	19	334.941	339.272	+ 4.331	340.522	−1.250	1.4106	331.682	−0.562
						Mean number	1.4152	332.244	

The preceding table shows how very near M. Dulong's value of K agrees with the result of experiment. Employing MM. Gay-Lussac's and Welter's co-efficient, the differences between the observed and calculated velocity are constantly affected with the same sign; whereas in taking K from M. Dulong, the differences are sometimes positive and sometimes negative. It is therefore presumed that M. Dulong's labours bring the calculation of the velocity of sound much nearer to the truth than before, and that such differences as are yet remaining between calculation and experiment, may be attributed, with great probability, to errors unavoidable in such complicated observations.

Utrecht, December 30, 1829.